

Boolean Algebra

Introduction

The most obvious way to simplify *boolean expressions* is to manipulate them in the same way as normal algebraic expressions are manipulated. With regards to logic relations in digital forms, a set of rules for symbolic manipulation is needed in order to solve for the unknowns.


A set of rules formulated by the English mathematician  **George Boole** describe certain propositions whose outcome would be either *true* or *false*. With regard to digital logic, these rules are used to describe circuits whose state can be either, *1 (true)* or *0 (false)*. In order to fully understand this, the relation between the AND gate, OR gate and NOT gate operations should be appreciated. A number of rules can be derived from these relations as Table 1 demonstrates.

Table 1: Boolean postulates

P1	$X = 0, X = 1$
P2	$0 \cdot 0 = 0$
P3	$1 + 1 = 1$
P4	$0 + 0 = 0$
P5	$1 \cdot 1 = 1$
P6	$1 \cdot 0 = 0 \cdot 1 = 0$
P7	$1 + 0 = 0 + 1 = 1$

Laws of Boolean Algebra

Table 2 shows the basic Boolean laws. Note that every law has two expressions, **a** and **b**. This is known as *duality*. These are obtained by changing every AND (\cdot) to OR ($+$), every OR to AND and all 1's to 0's and vice-versa.

Table 2: Boolean laws		
L1	Commutative law	a $A + B = B + A$
		b $A \cdot B = B \cdot A$
L2	Associative Law	a $(A + B) + C = A + (B + C)$
		b $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
L3	Distributive Law	a $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
		b $A + (B \cdot C) = (A + B) \cdot (A + C)$
L4	Identity Law	a $A + A = A$
		b $A \cdot A = A$
L5	...	a $(A \cdot B) + (A \cdot B) = A$
		b $(A + B) \cdot (A + B) = A$
L6	Redundancy Law	a $A + (A \cdot B) = A$
		b $A \cdot (A + B) = A$
L7	...	a $0 + A = A$
		b $0 \cdot A = 0$

Table 2: Boolean laws		
L8	...	a $1 + A = 1$
		b $1 \cdot A = A$
L9	...	a $!A + A = 1$
		b $!A \cdot A = 0$
L10	...	a $A + (!A \cdot B) = A + B$
		b $A \cdot (!A + B) = A \cdot B$
L11	De Morgan's Theorem	a $!(A + B) = !A \cdot !B$
		b $!(A \cdot B) = !A + !B$



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