

# Boolean Algebra

## Introduction

The most obvious way to simplify *boolean expressions* is to manipulate them in the same way as normal algebraic expressions are manipulated. With regards to logic relations in digital forms, a set of rules for symbolic manipulation is needed in order to solve for the unknowns.

A set of rules formulated by the English mathematician [George Boole](#) describe certain propositions whose outcome would be either *true* or *false*. With regard to digital logic, these rules are used to describe circuits whose state can be either, 1 (*true*) or 0 (*false*). In order to fully understand this, the relation between the AND gate, OR gate and NOT gate operations should be appreciated. A number of rules can be derived from these relations as Table 1 demonstrates.

**Table 1: Boolean postulates**

P1	$X = 0, X = 1$
P2	$0 \cdot 0 = 0$
P3	$1 + 1 = 1$
P4	$0 + 0 = 0$
P5	$1 \cdot 1 = 1$
P6	$1 \cdot 0 = 0 \cdot 1 = 0$
P7	$1 + 0 = 0 + 1 = 1$

## Laws of Boolean Algebra

Table 2 shows the basic Boolean laws. Note that every law has two expressions, **a** and **b**. This is known as *duality*. These are obtained by changing every AND ( $\cdot$ ) to OR ( $+$ ), every OR to AND and all 1's to 0's and vice-versa.

**Table 2: Boolean laws**

<b>L1</b>	Commutative law	<b>a</b>	$A + B = B + A$
		<b>b</b>	$A \cdot B = B \cdot A$
<b>L2</b>	Associative Law	<b>a</b>	$(A + B) + C = A + (B + C)$
		<b>b</b>	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$
<b>L3</b>	Distributive Law	<b>a</b>	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
		<b>b</b>	$A + (B \cdot C) = (A + B) \cdot (A + C)$
<b>L4</b>	Identity Law	<b>a</b>	$A + A = A$
		<b>b</b>	$A \cdot A = A$
<b>L5</b>	...	<b>a</b>	$(A \cdot B) + (A \cdot B) = A$
		<b>b</b>	$(A + B) \cdot (A + B) = A$
<b>L6</b>	Redundancy Law	<b>a</b>	$A + (A \cdot B) = A$
		<b>b</b>	$A \cdot (A + B) = A$
<b>L7</b>	...	<b>a</b>	$0 + A = A$
		<b>b</b>	$0 \cdot A = 0$

**Table 2: Boolean laws**

<b>L8</b> ...	<b>a</b>	$1 + A = 1$
	<b>b</b>	$1 \cdot A = A$
<b>L9</b> ...	<b>a</b>	$!A + A = 1$
	<b>b</b>	$!A \cdot A = 0$
<b>L10</b> ...	<b>a</b>	$A + (!A \cdot B) = A + B$
	<b>b</b>	$A \cdot (!A + B) = A \cdot B$
<b>L11</b> De Morgan's Theorem	<b>a</b>	$!(A + B) = !A \cdot !B$
	<b>b</b>	$!(A \cdot B) = !A + !B$



**XS Wiki** - <https://www.siscoxs.it/wiki/>

Link permanente:  
<https://www.siscoxs.it/wiki/note:booleanalgebra>

Ultimo aggiornamento: **2018-04-22 19:55**

