Boolean Algebra

Introduction

The most obvious way to simplify *boolean expressions* is to manipulate them in the same way as normal algebraic expressions are manipulated. With regards to logic relations in digital forms, a set of rules for symbolic manipulation is needed in order to solve for the unknowns.

A set of rules formulated by the English mathematician George Boole describe certain propositions whose outcome would be either *true* or *false*. With regard to digital logic, these rules are used to describe circuits whose state can be either, 1 (*true*) or 0 (*false*). In order to fully understand this, the relation between the AND gate, OR gate and NOT gate operations should be appreciated. A number of rules can be derived from these relations as Table 1 demonstrates.

Table 1: Boolean postulates					
P1	X = 0, X = 1				
P2	$0 \cdot 0 = 0$				
Р3	1 + 1 = 1				
P4	0 + 0 = 0				
P5	$1 \cdot 1 = 1$				
P6	$1 \cdot 0 = 0 \cdot 1 = 0$				
P7	1 + 0 = 0 + 1 = 1				

Laws of Boolean Algebra

Table 2 shows the basic Boolean laws. Note that every law has two expressions, \boldsymbol{a} and \boldsymbol{b} . This is known as *duality*. These are obtained by changing every AND (·) to OR (+), every OR to AND and all 1's to 0's and vice-versa.

Table 2: Boolean laws				
L1	Commutative law	a	A + B = B + A	
		b	$A \cdot B = B \cdot A$	
L2	Associative Law ⊢	a	(A + B) + C = A + (B + C)	
		b	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$	
L3	Distributive Law		$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$	
	Distributive raw	b	$A + (B \cdot C) = (A + B) \cdot (A + C)$	
L4	Identity Law	a	A + A = A	
	Identity Law		$A \cdot A = A$	
L5	l -		$(A \cdot B) + (A \cdot B) = A$	
			$(A + B) \cdot (A + B) = A$	
L6	Redundancy Law ⊢	a	$A + (A \cdot B) = A$	
		b	$A \cdot (A + B) = A$	
L7		a	0 + A = A	
			$0 \cdot A = 0$	

Table 2: Boolean laws				
L8		a	1 + A = 1	
		b	$1 \cdot A = A$	
L9		a	A + A = 1	
		b	$ A\cdot A = 0$	
L10		a	$A + (!A \cdot B) = A + B$	
		b	$A \cdot (!A + B) = A \cdot B$	
L11	De Morgan's Theorem	a	$!(A + B) = !A \cdot ! B$	
		b	$!(A \cdot B) = !A + !B$	



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