Boolean Algebra

Introduction

The most obvious way to simplify *boolean expressions* is to manipulate them in the same way as normal algebraic expressions are manipulated. With regards to logic relations in digital forms, a set of rules for symbolic manipulation is needed in order to solve for the unknowns.

A set of rules formulated by the English mathematician George Boole describe certain propositions whose outcome would be either *true* or *false*. With regard to digital logic, these rules are used to describe circuits whose state can be either, *1 (true)* or *0 (false)*. In order to fully understand this, the relation between the `AND` gate, `OR` gate and `NOT` gate operations should be appreciated. A number of rules can be derived from these relations as Table 1 demonstrates.

Table 1: Boolean postulates	
P1	$X = 0, X = 1$
P2	$0 \cdot 0 = 0$
P3	$1 + 1 = 1$
P4	$0 + 0 = 0$
P5	$1 \cdot 1 = 1$
P6	$1 \cdot 0 = 0 \cdot 1 = 0$
P7	$1 + 0 = 0 + 1 = 1$

Laws of Boolean Algebra

Table 2 shows the basic Boolean laws. Note that every law has two expressions, ***a*** and ***b***. This is known as *duality*. These are obtained by changing every `AND` (·) to `OR` (+), every `OR` to `AND` and all 1's to 0's and vice-versa.

Table 2: Boolean laws			
L1	Commutative law	***a***	$A + B = B + A$
		b	$A \cdot B = B \cdot A$
L2	Associative Law	***a***	$(A + B) + C = A + (B + C)$
		b	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$
L3	Distributive Law	***a***	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
		b	$A + (B \cdot C) = (A + B) \cdot (A + C)$
L4	Identity Law	***a***	$A + A = A$
		b	$A \cdot A = A$
L5	…	***a***	$(A \cdot B) + (A \cdot B) = A$
		b	$(A + B) \cdot (A + B) = A$
L6	Redundancy Law	***a***	$A + (A \cdot B) = A$
		b	$A \cdot (A + B) = A$
L7	…	***a***	$0 + A = A$
		b	$0 \cdot A = 0$

Table 2: Boolean laws			
L8	…	***a***	1 + A = 1
		b	1 · A = A
L9	…	***a***	!A + A = 1
		b	!A · A = 0
L10	…	***a***	A + (!A · B) = A + B
		b	A · (!A + B) = A · B
L11	De Morgan's Theorem	***a***	!(A + B) = !A ·! B
		b	!(A · B) = !A + !B

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