


# Boolean Algebra

## Introduction

The most obvious way to simplify *boolean expressions* is to manipulate them in the same way as normal algebraic expressions are manipulated. With regards to logic relations in digital forms, a set of rules for symbolic manipulation is needed in order to solve for the unknowns.

A set of rules formulated by the English mathematician  **George Boole** describe certain propositions whose outcome would be either *true* or *false*. With regard to digital logic, these rules are used to describe circuits whose state can be either, *1 (true)* or *0 (false)*. In order to fully understand this, the relation between the AND gate, OR gate and NOT gate operations should be appreciated. A number of rules can be derived from these relations as Table 1 demonstrates.

**Table 1: Boolean postulates**

P1	$X = 0, X = 1$
P2	$0 \cdot 0 = 0$
P3	$1 + 1 = 1$
P4	$0 + 0 = 0$
P5	$1 \cdot 1 = 1$
P6	$1 \cdot 0 = 0 \cdot 1 = 0$
P7	$1 + 0 = 0 + 1 = 1$

## Laws of Boolean Algebra

Table 2 shows the basic Boolean laws. Note that every law has two expressions, **a** and **b**. This is known as *duality*. These are obtained by changing every AND ( $\cdot$ ) to OR ( $+$ ), every OR to AND and all 1's to 0's and vice-versa.

Table 2: Boolean laws		
L1	Commutative law	<b>a</b> $A + B = B + A$
		<b>b</b> $A \cdot B = B \cdot A$
L2	Associative Law	<b>a</b> $(A + B) + C = A + (B + C)$
		<b>b</b> $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
L3	Distributive Law	<b>a</b> $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
		<b>b</b> $A + (B \cdot C) = (A + B) \cdot (A + C)$
L4	Identity Law	<b>a</b> $A + A = A$
		<b>b</b> $A \cdot A = A$
L5	...	<b>a</b> $(A \cdot B) + (A \cdot B) = A$
		<b>b</b> $(A + B) \cdot (A + B) = A$
L6	Redundancy Law	<b>a</b> $A + (A \cdot B) = A$
		<b>b</b> $A \cdot (A + B) = A$
L7	...	<b>a</b> $0 + A = A$
		<b>b</b> $0 \cdot A = 0$

Table 2: Boolean laws		
<b>L8</b>	...	<b>a</b> $1 + A = 1$
		<b>b</b> $1 \cdot A = A$
<b>L9</b>	...	<b>a</b> $!A + A = 1$
		<b>b</b> $!A \cdot A = 0$
<b>L10</b>	...	<b>a</b> $A + (!A \cdot B) = A + B$
		<b>b</b> $A \cdot (!A + B) = A \cdot B$
<b>L11</b>	De Morgan's Theorem	<b>a</b> $!(A + B) = !A \cdot !B$
		<b>b</b> $!(A \cdot B) = !A + !B$



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[www.ee.surrey.ac.uk/Projects/Labview/boolalgrebra](http://www.ee.surrey.ac.uk/Projects/Labview/boolalgrebra) — now on web archive)

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